IBO

DIPLOMA PROGRAMME

MATHEMATICS SL

International Baccalaureate

Syllabus & Assessment Information

Name: ____________________

Table of Contents:

Syllabus content

Topic 1—Algebra 8 hrs

Aims

The aim of this section is to introduce students to some basic algebraic concepts and applications. Number systems are now in the presumed knowledge section.

Details

	Content	Amplifications/inclusions	Exclusions
1.1	Arithmetic sequences and series; sum of finite arithmetic series; geometric sequences and series; sum of finite and infinite geometric series. Sigma notation.	Examples of applications, compound interest and population growth.	
1.2	Exponents and logarithms. Laws of exponents; laws of logarithms. Change of base.	Elementary treatment only is required. Examples: $16^{\frac{3}{4}}=8$; $\frac{3}{4}=\log_{16}8$; $\log 32=5\log 2$; $(2^3)^{-4}=2^{-12}$. $\log_b a=\frac{\log_c a}{\log_c b}$.	
1.3	The binomial theorem: expansion of $(a+b)^n$, $n\in\mathbb{N}$.	On examination papers: students may determine the binomial coefficients, $\binom{n}{r}$, by using Pascal's triangle, or by using a GDC.	The formula $\binom{n}{r}=\frac{n!}{r!(n-r)!}$ and consideration of combinations.

Topic 2—Functions and equations 24 hrs

Aims

The aims of this section are to explore the notion of function as a unifying theme in mathematics, and to apply functional methods to a variety of mathematical situations. It is expected that extensive use will be made of a GDC in both the development and the application of this topic.

Details

	Content	Amplifications/inclusions	Exclusions
2.1	Concept of function $f: x \mapsto f(x)$: domain, range; image (value).	On examination papers: if the domain is the set of real numbers then the statement "$x \in \mathbb{R}$" will be omitted.	Formal definition of a function; the terms "one-to-one", "many-to-one" and "codomain".
	Composite functions $f \circ g$; identity function.	The composite function $(f \circ g)(x)$ is defined as $f(g(x))$.	
	Inverse function f^{-1}.	On examination papers: if an inverse function is to be found, the given function will be defined with a domain that ensures it is one-to-one.	Domain restriction.
2.2	The graph of a function; its equation $y = f(x)$.	On examination papers: questions may be set requiring the graphing of functions that do not explicitly appear on the syllabus. The linear function $ax + b$ is now in the presumed knowledge section.	
	Function graphing skills:		
	use of a GDC to graph a variety of functions;		
	investigation of key features of graphs.	Identification of horizontal and vertical asymptotes.	
	Solution of equations graphically.	May be referred to as roots of equations, or zeros of functions.	

Topic 2—Functions and equations (continued)

	Content	Amplifications/inclusions	Exclusions
2.3	Transformations of graphs: translations; stretches; reflections in the axes.	Translations: $y = f(x) + b$; $y = f(x - a)$. Stretches: $y = pf(x)$; $y = f\left(x/q\right)$. Reflections (in both axes): $y = -f(x)$; $y = f(-x)$. Examples: $y = x^2$ used to obtain $y = 3x^2 + 2$ by a stretch of scale factor 3 in the y-direction followed by a translation of $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$. $y = \sin x$ used to obtain $y = 3\sin 2x$ by a stretch of scale factor 3 in the y-direction and a stretch of scale factor $\frac{1}{2}$ in the x-direction.	
	The graph of $y = f^{-1}(x)$ as the reflection in the line $y = x$ of the graph of $y = f(x)$.		
2.4	The reciprocal function $x \mapsto \frac{1}{x}$, $x \neq 0$: its graph; its self-inverse nature.		

	Content	Amplifications/inclusions	Exclusions
2.5	The quadratic function $x \mapsto ax^2 + bx + c$: its graph, y-intercept $(0, c)$. Axis of symmetry $x = -\frac{b}{2a}$. The form $x \mapsto a(x-h)^2 + k$: vertex (h, k). The form $x \mapsto a(x-p)(x-q)$: x-intercepts $(p, 0)$ and $(q, 0)$.	Rational coefficients only.	
2.6	The solution of $ax^2 + bx + c = 0$, $a \neq 0$. The quadratic formula. Use of the discriminant $\Delta = b^2 - 4ac$.		On examination papers: questions demanding elaborate factorization techniques will not be set.
2.7	The function: $x \mapsto a^x$, $a > 0$. The inverse function $x \mapsto \log_a x$, $x > 0$. Graphs of $y = a^x$ and $y = \log_a x$. Solution of $a^x = b$ using logarithms.	$\log_a a^x = x$; $a^{\log_a x} = x$, $x > 0$.	
2.8	The exponential function $x \mapsto \mathrm{e}^x$. The logarithmic function $x \mapsto \ln x$, $x > 0$.	$a^x = e^{x \ln a}$. Examples of applications: compound interest, growth and decay.	

Topic 3—Circular functions and trigonometry 16 hrs

Aims

The aims of this section are to explore the circular functions and to solve triangles using trigonometry.

Details

	Content	Amplifications/inclusions	Exclusions
3.1	The circle: radian measure of angles; length of an arc; area of a sector.	Radian measure may be expressed as multiples of π, or decimals.	
3.2	Definition of $\cos\theta$ and $\sin\theta$ in terms of the unit circle. Definition of $\tan\theta$ as $\dfrac{\sin\theta}{\cos\theta}$. The identity $\cos^2\theta+\sin^2\theta=1$.	Given $\sin\theta$, finding possible values of $\cos\theta$ without finding θ. Lines through the origin can be expressed as $y=x\tan\theta$, with gradient $\tan\theta$.	The reciprocal trigonometric functions $\sec\theta$, $\csc\theta$ and $\cot\theta$.
3.3	Double angle formulae: $\sin 2\theta=2\sin\theta\cos\theta$; $\cos 2\theta=\cos^2\theta-\sin^2\theta$.	Double angle formulae can be established by simple geometrical diagrams and/or by use of a GDC.	Compound angle formulae.
3.4	The circular functions $\sin x$, $\cos x$ and $\tan x$: their domains and ranges; their periodic nature; and their graphs. Composite functions of the form $f(x)=a\sin(b(x+c))+d$.	On examination papers: radian measure should be assumed unless otherwise indicated by, for example, $x\mapsto\sin x°$. Example: $f(x)=2\cos(3(x-4))+1$. Examples of applications: height of tide, Ferris wheel.	The inverse trigonometric functions: $\arcsin x$, $\arccos x$ and $\arctan x$.

Topic 3—Circular functions and trigonometry (continued)

	Content	Amplifications/inclusions	Exclusions
3.5	Solution of trigonometric equations in a finite interval.	Examples: $2\sin x = 3\cos x$, $0 \le x \le 2\pi$. $2\sin 2x = 3\cos x$, $0^\circ \le x \le 180^\circ$. $2\sin x = \cos 2x$, $-\pi \le x \le \pi$. Both analytical and graphical methods required.	The general solution of trigonometric equations.
	Equations of the type $a\sin(b(x+c)) = k$.		
	Equations leading to quadratic equations in, for example, $\sin x$.		
	Graphical interpretation of the above.		
3.6	Solution of triangles.		
	The cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$.	Appreciation of Pythagoras' theorem as a special case of the cosine rule.	
	The sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.	The ambiguous case of the sine rule.	
	Area of a triangle as $\dfrac{1}{2}ab\sin C$.		
		Applications to problems in real-life situations, such as navigation.	

Topic 4—Matrices 10 hrs

Aims

The aim of this section is to provide an elementary introduction to matrices, a fundamental concept of linear algebra.

Details

	Content	Amplifications/inclusions	Exclusions
4.1	Definition of a matrix: the terms “element”, “row”, “column” and “order”.	Use of matrices to store data.	Use of matrices to represent transformations.
4.2	Algebra of matrices: equality; addition; subtraction; multiplication by a scalar. Multiplication of matrices. Identity and zero matrices.	Matrix operations to handle or process information.	
4.3	Determinant of a square matrix. Calculation of 2×2 and 3×3 determinants. Inverse of a 2×2 matrix. Conditions for the existence of the inverse of a matrix.	Elementary treatment only. Obtaining the inverse of a 3×3 matrix using a GDC.	 Cofactors and minors. Other methods for finding the inverse of a 3×3 matrix.
4.4	Solution of systems of linear equations using inverse matrices (a maximum of three equations in three unknowns).	Only systems with a unique solution need be considered.	

Topic 5—Vectors 16 hrs

Aims

The aim of this section is to provide an elementary introduction to vectors. This includes both algebraic and geometric approaches.

Details

	Content	Amplifications/inclusions	Exclusions
5.1	Vectors as displacements in the plane and in three dimensions.	Distance between points in three dimensions.	
	Components of a vector; column representation. $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1\boldsymbol{i} + v_2\boldsymbol{j} + v_3\boldsymbol{k}$.	Components are with respect to the unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ (standard basis).	
	Algebraic and geometric approaches to the following topics:		
	the sum and difference of two vectors; the zero vector, the vector $-\boldsymbol{v}$;	The difference of $\boldsymbol{v}$ and $\boldsymbol{w}$ is $\boldsymbol{v} - \boldsymbol{w} = \boldsymbol{v} + (-\boldsymbol{w})$.	
	multiplication by a scalar, $k\boldsymbol{v}$;		
	magnitude of a vector, $\lvert\boldsymbol{v}\rvert$;		
	unit vectors; base vectors $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$;		
	position vectors $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$.	$\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}} = \boldsymbol{b} - \boldsymbol{a}$.	

Topic 5—Vectors (continued)

	Content	Amplifications/inclusions	Exclusions
5.2	The scalar product of two vectors $\boldsymbol{v}\cdot\boldsymbol{w}=\lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert\cos\theta$; $\boldsymbol{v}\cdot\boldsymbol{w}=v_1w_1+v_2w_2+v_3w_3$. Perpendicular vectors; parallel vectors. The angle between two vectors.	The scalar product is also known as the "dot product" or "inner product". For non-zero perpendicular vectors $\boldsymbol{v}\cdot\boldsymbol{w}=0$; for non-zero parallel vectors $\boldsymbol{v}\cdot\boldsymbol{w}=\pm\lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert$.	Projections.
5.3	Representation of a line as $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$. The angle between two lines.	Lines in the plane and in three-dimensional space. Examples of applications: interpretation of t as time and $\boldsymbol{b}$ as velocity, with $\lvert\boldsymbol{b}\rvert$ representing speed.	Cartesian form of the equation of a line: $\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n}$.
5.4	Distinguishing between coincident and parallel lines. Finding points where lines intersect.	Awareness that non-parallel lines may not intersect.	

Topic 6—Statistics and probability 30 hrs

Aims

The aim of this section is to introduce basic concepts. It may be considered as three parts: descriptive statistics (6.1–6.4), basic probability (6.5–6.8), and modelling data (6.9–6.11). It is expected that most of the calculations required will be done on a GDC. The emphasis is on understanding and interpreting the results obtained.

Details

	Content	Amplifications/inclusions	Exclusions
6.1	Concepts of population, sample, random sample and frequency distribution of discrete and continuous data.	Elementary treatment only.	
6.2	Presentation of data: frequency tables and diagrams, box and whisker plots. Grouped data: mid-interval values, interval width, upper and lower interval boundaries, frequency histograms.	Treatment of both continuous and discrete data. A frequency histogram uses equal class intervals.	 Histograms based on unequal class intervals.

Topic 6—Statistics and probability (continued)

	Content	Amplifications/inclusions	Exclusions
6.3	Mean, median, mode; quartiles, percentiles.	Awareness that the population mean, μ, is generally unknown, and that the sample mean, $\overline{x}$, serves as an estimate of this quantity.	Estimation of the mode from a histogram.
	Range; interquartile range; variance; standard deviation.	Awareness of the concept of dispersion and an understanding of the significance of the numerical value of the standard deviation.	
		Obtaining the standard deviation (and indirectly the variance) from a GDC is expected.	Other methods for finding the standard deviation or variance.
		Awareness that the population standard deviation, σ, is generally unknown, and that the standard deviation of the sample, s_n, serves as an estimate of this quantity.	Discussion of bias of s_n^2 as an estimate of σ^2.
6.4	Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles.		
6.5	Concepts of trial, outcome, equally likely outcomes, sample space (U) and event.		
	The probability of an event A as $\mathrm{P}(A)=\dfrac{n(A)}{n(U)}$.		
	The complementary events A and A' (not A); $\mathrm{P}(A)+\mathrm{P}(A')=1$.		

Topic 6—Statistics and probability (continued)

	Content	Amplifications/inclusions	Exclusions
6.6	Combined events, the formula: $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$. $\mathrm{P}(A \cap B) = 0$ for mutually exclusive events.	Appreciation of the non-exclusivity of “or”. Use of $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$ for mutually exclusive events.	
6.7	Conditional probability; the definition $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$. Independent events; the definition $\mathrm{P}(A \mid B) = \mathrm{P}(A) = \mathrm{P}(A \mid B')$.	The term “independent” is equivalent to “statistically independent”. Use of $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$ for independent events.	
6.8	Use of Venn diagrams, tree diagrams and tables of outcomes to solve problems.		
6.9	Concept of discrete random variables and their probability distributions. Expected value (mean), $\mathrm{E}(X)$ for discrete data.	Simple examples only, such as: $\mathrm{P}(X = x) = \dfrac{1}{18}(4 + x)$ for $x \in \{1, 2, 3\}$; $\mathrm{P}(X = x) = \dfrac{5}{18}, \dfrac{6}{18}, \dfrac{7}{18}$. Knowledge and use of the formula $\mathrm{E}(X) = \sum \left(x\mathrm{P}(X = x) \right)$. Applications of expectation, for example, games of chance.	Formal treatment of random variables and probability density functions; formal treatment of cumulative frequency distributions and their formulae.

	Content	Amplifications/inclusions	Exclusions
6.10	Binomial distribution.		The formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ and consideration of combinations.
	Mean of the binomial distribution.		Formal proof of mean.
6.11	Normal distribution.		Normal approximation to the binomial distribution.
	Properties of the normal distribution.	Appreciation that the standardized value (z) gives the number of standard deviations from the mean.	
	Standardization of normal variables.	Use of calculator (or tables) to find normal probabilities; the reverse process.	

Topic 7—Calculus 36 hrs

Aims

The aim of this section is to introduce students to the basic concepts and techniques of differential and integral calculus and their application.

Details

	Content	Amplifications/inclusions	Exclusions
7.1	Informal ideas of limit and convergence.	Only an informal treatment of limit and convergence, for example, 0.3, 0.33, 0.333, ... converges to $\frac{1}{3}$.	
	Definition of derivative as $f'(x)=\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$.	Use of this definition for derivatives of polynomial functions only. Other derivatives can be justified by graphical considerations using a GDC. Familiarity with both forms of notation, $\frac{dy}{dx}$ and $f'(x)$, for the first derivative.	
	Derivative of x^n $(n\in\mathbb{Q})$, $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$.		
	Derivative interpreted as gradient function and as rate of change.	Finding equations of tangents and normals. Identifying increasing and decreasing functions.	

	Content	Amplifications/inclusions	Exclusions
7.2	Differentiation of a sum and a real multiple of the functions in 7.1. The chain rule for composite functions. The product and quotient rules. The second derivative.	Familiarity with both forms of notation, $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ and $f''(x)$, for the second derivative.	
7.3	Local maximum and minimum points. Use of the first and second derivative in optimization problems.	Testing for maximum or minimum using change of sign of the first derivative and using sign of the second derivative. Examples of applications: profit, area, volume.	
7.4	Indefinite integration as anti-differentiation. Indefinite integral of x^n $(n \in \mathbb{Q})$, $\sin x$, $\cos x$, $\frac{1}{x}$ and e^x. The composites of any of these with the linear function $ax+b$.	$\int \frac{1}{x}\mathrm{d}x = \ln x + C$, $x > 0$. Example: $f'(x) = \cos(2x+3) \Rightarrow f(x) = \frac{1}{2}\sin(2x+3) + C$.	

	Content	Amplifications/inclusions	Exclusions
7.5	Anti-differentiation with a boundary condition to determine the constant term.	Example: if $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + x$ and $y = 10$ when $x = 0$, then $y = x^3 + \frac{1}{2}x^2 + 10$.	
	Definite integrals.		
	Areas under curves (between the curve and the x-axis), areas between curves.	Only the form $\int_a^b y\,\mathrm{d}x$.	$\int_a^b x\,\mathrm{d}y$.
	Volumes of revolution.	Revolution about the x-axis only, $V = \int_a^b \pi y^2\,\mathrm{d}x$.	Revolution about the y-axis; $V = \int_a^b \pi x^2\,\mathrm{d}y$.
7.6	Kinematic problems involving displacement, s, velocity, v, and acceleration, a.	$v = \frac{\mathrm{d}s}{\mathrm{d}t}$, $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$. Area under velocity–time graph represents distance.	
7.7	Graphical behaviour of functions: tangents and normals, behaviour for large $\lvert x \rvert$,	Both “global” and “local” behaviour.	
	horizontal and vertical asymptotes.		Oblique asymptotes.
	The significance of the second derivative; distinction between maximum and minimum points.	Use of the terms “concave-up” for $f''(x) > 0$, “concave-down” for $f''(x) < 0$.	
	Points of inflexion with zero and non-zero gradients.	At a point of inflexion $f''(x) = 0$ and $f''(x)$ changes sign (concavity change). $f''(x) = 0$ is not a sufficient condition for a point of inflexion: for example, $y = x^4$ at $(0,0)$.	Points of inflexion where $f''(x)$ is not defined: for example, $y = x^{1/3}$ at $(0,0)$.

ASSESSMENT OUTLINE

First examinations 2008

Mathematics SL

External assessment	**3 hrs**	**80%**
Written papers		
Paper 1	**1 hr 30 min**	**40%**
No calculator allowed		
Section A		20%
Compulsory short-response questions based on the whole syllabus		
Section B		20%
Compulsory extended-response questions based on the whole syllabus		
Paper 2	**1 hr 30 min**	**40%**
Graphic display calculator (GDC) required		
Section A		20%
Compulsory short-response questions based on the whole syllabus		
Section B		20%
Compulsory extended-response questions based on the whole syllabus		

ASSESSMENT DETAILS

External assessment details 3 hrs 80%

General

Paper 1 and paper 2

These papers are externally set and externally marked. Together they contribute 80% of the final mark for the course. These papers are designed to allow students to demonstrate what they know and what they can do.

Calculators

Paper 1

Students are not permitted access to any calculator. Questions will mainly involve analytic approaches to solutions, rather than requiring the use of a GDC. It is not intended to have complicated calculations, with the potential for careless errors. However, questions will include some arithmetical manipulations when they are essential to the development of the question.

Paper 2

Students must have access to a GDC at all times. However, not all questions will necessarily require the use of the GDC. Regulations covering the types of GDC allowed are provided in the *Vade Mecum*.

Mathematics SL information booklet

Each student must have access to a clean copy of the information booklet during the examination.

Awarding of marks

Marks may be awarded for method, accuracy, answers and reasoning, including interpretation.

In paper 1 and paper 2, full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations (in the form of, for example, diagrams, graphs or calculations). Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. All students should therefore be advised to show their working.

Paper 1 1 hr 30 min 40%

This paper consists of section A, short-response questions, and section B, extended-response questions. Each section will be worth 20% of the total mark. Students are not permitted access to any calculator on this paper.

Syllabus coverage

- Knowledge of **all** topics is required for this paper. However, not all topics are necessarily assessed in every examination session.

Mark allocation

- This paper is worth **90** marks, representing **40%** of the final mark.
- Questions of varying levels of difficulty and length are set. Therefore, individual questions may not necessarily be worth the same number of marks. The exact number of marks allocated to each question is indicated at the start of the question.

Section A

This section consists of compulsory short-response questions based on the whole syllabus. It is worth 45 marks, representing 20% of the final mark.

- The intention of this section is to test students' knowledge across the breadth of the syllabus. However, it should not be assumed that the separate topics are given equal emphasis.

Question type

- A small number of steps is needed to solve each question.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.

Section B

This section consists of a small number of compulsory extended-response questions based on the whole syllabus. It is worth 45 marks, representing 20% of the final mark.

- Individual questions may require knowledge of more than one topic.
- The intention of this section is to test students' knowledge of the syllabus in depth. The range of syllabus topics tested in this paper may be narrower than that tested in section A.

Question type

- Questions require extended responses involving sustained reasoning.
- Individual questions may develop a single theme or be divided into unconnected parts.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.
- Normally, each question reflects an incline of difficulty, from relatively easy tasks at the start of a question to relatively difficult tasks at the end of a question. The emphasis is on problem-solving.

Paper 2 1 hr 30 min 40%

This paper consists of section A, short-response questions, and section B, extended-response questions. Each section will be worth 20% of the total mark. A GDC is required for this paper, but not every question will necessarily require its use.

Syllabus coverage

- Knowledge of **all** topics is required for this paper. However, not all topics are necessarily assessed in every examination session.

Mark allocation

- This paper is worth **90** marks, representing **40%** of the final mark.
- Questions of varying levels of difficulty and length are set. Therefore, individual questions may not necessarily be worth the same number of marks. The exact number of marks allocated to each question is indicated at the start of the question.

Section A

This section consists of compulsory short-response questions based on the whole syllabus. It is worth 45 marks, representing 20% of the final mark.

- The intention of this section is to test students' knowledge across the breadth of the syllabus. However, it should not be assumed that the separate topics are given equal emphasis.

Question type

- A small number of steps is needed to solve each question.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.

Section B

This section consists of a small number of compulsory extended-response questions based on the whole syllabus. It is worth 45 marks, representing 20% of the final mark.

- Individual questions may require knowledge of more than one topic.
- The intention of this section is to test students' knowledge of the syllabus in depth. The range of syllabus topics tested in this paper may be narrower than that tested in section A.

Question type

- Questions require extended responses involving sustained reasoning.
- Individual questions may develop a single theme or be divided into unconnected parts.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.
- Normally, each question reflects an incline of difficulty, from relatively easy tasks at the start of a question to relatively difficult tasks at the end of a question. The emphasis is on problem-solving.

Guidelines

Notation

Of the various notations in use, the IBO has chosen to adopt a system of notation based on the recommendations of the International Organization for Standardization (ISO). This notation is used in the examination papers for this course without explanation. If forms of notation other than those listed in this guide are used on a particular examination paper, they are defined within the question in which they appear.

Because students are required to recognize, though not necessarily use, IBO notation in examinations, it is recommended that teachers introduce students to this notation at the earliest opportunity. Students are **not** allowed access to information about this notation in the examinations.

In a small number of cases, students may need to use alternative forms of notation in their written answers. This is because not all forms of IBO notation can be directly transferred into handwritten form. For vectors in particular the IBO notation uses a bold, italic typeface that cannot adequately be transferred into handwritten form. In this case, teachers should advise students to use alternative forms of notation in their written work (for example, $\vec{x}$, $\bar{x}$ or $\underline{x}$).

Students must always use correct mathematical notation, not calculator notation.

$\mathbb{N}$	the set of positive integers and zero, $\{0, 1, 2, 3, ...\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, ...\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, ...\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \mid x \in \mathbb{Q}, x > 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \mid x \in \mathbb{R}, x > 0\}$
$\{x_1, x_2, ...\}$	the set with elements $x_1, x_2, ...$
$n(A)$	the number of elements in the finite set A
$\{x \mid \quad \}$	the set of all x such that
$\in$	is an element of
$\notin$	is not an element of
$\varnothing$	the empty (null) set
U	the universal set
$\cup$	union
$\cap$	intersection
$\subset$	is a proper subset of
$\subseteq$	is a subset of

A'	the complement of the set A
$a \mid b$	a divides b
$a^{1/n}$, $\sqrt[n]{a}$	a to the power of $\frac{1}{n}$, n^{th} root of a (if $a \geq 0$ then $\sqrt[n]{a} \geq 0$)
$a^{1/2}$, $\sqrt{a}$	a to the power $\frac{1}{2}$, square root of a (if $a \geq 0$ then $\sqrt{a} \geq 0$)
$\lvert x \rvert$	the modulus or absolute value of x, that is $\begin{cases} x & \text{for } x \geq 0, x \in \mathbb{R} \\ -x & \text{for } x < 0, x \in \mathbb{R} \end{cases}$
$\approx$	is approximately equal to
$>$	is greater than
$\geq$	is greater than or equal to
$<$	is less than
$\leq$	is less than or equal to
$\ngtr$	is not greater than
$\nless$	is not less than
u_n	the n^{th} term of a sequence or series
d	the common difference of an arithmetic sequence
r	the common ratio of a geometric sequence
S_n	the sum of the first n terms of a sequence, $u_1 + u_2 + ... + u_n$
S_∞	the sum to infinity of a sequence, $u_1 + u_2 + ...$
$\sum_{i=1}^{n} u_i$	$u_1 + u_2 + ... + u_n$
$\prod_{i=1}^{n} u_i$	$u_1 \times u_2 \times ... \times u_n$
$\binom{n}{r}$	the r^{th} binomial coefficient, $r = 0, 1, 2, ...$, in the expansion of $(a+b)^n$
$f : A \to B$	f is a function under which each element of set A has an image in set B
$f : x \mapsto y$	f is a function under which x is mapped to y
$f(x)$	the image of x under the function f
f^{-1}	the inverse function of the function f

$f \circ g$	the composite function of f and g
$\lim_{x \to a} f(x)$	the limit of $f(x)$ as x tends to a
$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$f'(x)$	the derivative of $f(x)$ with respect to x
$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$	the second derivative of y with respect to x
$f''(x)$	the second derivative of $f(x)$ with respect to x
$\int y \, \mathrm{d}x$	the indefinite integral of y with respect to x
$\int_a^b y \, \mathrm{d}x$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
e^x	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$	the natural logarithm of x, $\log_{\mathrm{e}} x$
sin, cos, tan	the circular functions
$\mathrm{A}(x, y)$	the point A in the plane with Cartesian coordinates x and y
$[\mathrm{AB}]$	the line segment with end points A and B
AB	the length of $[\mathrm{AB}]$
(AB)	the line containing points A and B
$\hat{A}$	the angle at A
$\mathrm{C\hat{A}B}$	the angle between $[\mathrm{CA}]$ and $[\mathrm{AB}]$
$\Delta\mathrm{ABC}$	the triangle whose vertices are A, B and C
$\boldsymbol{v}$	the vector $\boldsymbol{v}$
$\vec{\mathrm{AB}}$	the vector represented in magnitude and direction by the directed line segment from A to B
$\boldsymbol{a}$	the position vector $\vec{\mathrm{OA}}$
$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$	unit vectors in the directions of the Cartesian coordinate axes
$\lvert \boldsymbol{a} \rvert$	the magnitude of $\boldsymbol{a}$

$\lvert\overrightarrow{\mathrm{AB}}\rvert$	the magnitude of $\overrightarrow{\mathrm{AB}}$
$\boldsymbol{v}\cdot\boldsymbol{w}$	the scalar product of $\boldsymbol{v}$ and $\boldsymbol{w}$
$\boldsymbol{A}^{-1}$	the inverse of the non-singular matrix $\boldsymbol{A}$
$\boldsymbol{A}^{\mathrm{T}}$	the transpose of the matrix $\boldsymbol{A}$
$\det \boldsymbol{A}$	the determinant of the square matrix $\boldsymbol{A}$
$\boldsymbol{I}$	the identity matrix
$\mathrm{P}(A)$	probability of event A
$\mathrm{P}(A')$	probability of the event "not A"
$\mathrm{P}(A \mid B)$	probability of the event A given B
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
$\mathrm{B}(n, p)$	binomial distribution with parameters n and p
$\mathrm{N}(\mu, \sigma^2)$	normal distribution with mean μ and variance σ^2
$X \sim \mathrm{B}(n, p)$	the random variable X has a binomial distribution with parameters n and p
$X \sim \mathrm{N}(\mu, \sigma^2)$	the random variable X has a normal distribution with mean μ and variance σ^2
μ	population mean
σ^2	population variance, $\sigma^2 = \dfrac{\sum_{i=1}^{k} f_i (x_i - \mu)^2}{n}$, where $n = \sum_{i=1}^{k} f_i$
σ	population standard deviation
$\bar{x}$	sample mean
s_n^2	sample variance, $s_n^2 = \dfrac{\sum_{i=1}^{k} f_i (x_i - \bar{x})^2}{n}$, where $n = \sum_{i=1}^{k} f_i$
s_n	standard deviation of the sample
Φ	cumulative distribution function of the standardized normal variable with distribution $\mathrm{N}(0,\ 1)$

Glossary of command terms

The following command terms are used without explanation on examination papers. Teachers should familiarize themselves and their students with the terms and their meanings. This list is not exhaustive. Other command terms may be used, but it should be assumed that they have their usual meaning (for example, "explain" and "estimate"). The terms included here are those that sometimes have a meaning in mathematics that is different from the usual meaning.

Further clarification and examples can be found in the teacher support material.

Write down	Obtain the answer(s), usually by extracting information. Little or no calculation is required. Working does not need to be shown.
Calculate	Obtain the answer(s) showing all relevant working. "Find" and "determine" can also be used.
Find	Obtain the answer(s) showing all relevant working. "Calculate" and "determine" can also be used.
Determine	Obtain the answer(s) showing all relevant working. "Find" and "calculate" can also be used.
Differentiate	Obtain the derivative of a function.
Integrate	Obtain the integral of a function.
Solve	Obtain the solution(s) or root(s) of an equation.
Draw	Represent by means of a labelled, **accurate** diagram or graph, using a pencil. A ruler (straight edge) should be used for straight lines. Diagrams should be drawn to scale. Graphs should have points correctly plotted (if appropriate) and joined in a straight line or smooth curve.
Sketch	Represent by means of a diagram or graph, labelled if required. A sketch should give a general idea of the required shape of the diagram or graph. A sketch of a graph should include relevant features such as intercepts, maxima, minima, points of inflexion and asymptotes.
Plot	Mark the position of points on a diagram.
Compare	Describe the similarities and differences between two or more items.
Deduce	Show a result using known information.
Justify	Give a valid reason for an answer or conclusion.
Show that	Obtain the required result (possibly using information given) without the formality of proof. "Show that" questions should not generally be "analysed" using a calculator.
Hence	Use the preceding work to obtain the required result.
Hence or otherwise	It is suggested that the preceding work is used, but other methods could also receive credit.