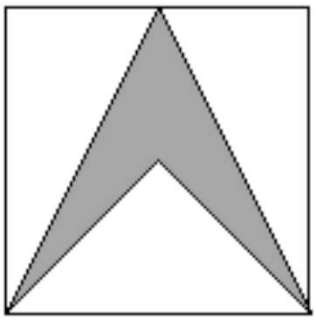
8 Questions Most High School Math Student Cannot Answer

1.* There are exactly three different pairs of positive integers that add to make six.

1+5=6 2+4=6 3+3=6

How many different pairs of positive integers add to make one-thousand?

2.* An arrow is formed in a 2 by 2 square by joining the bottom corners to the midpoint of the top edge and the centre of the square.



Find the area of the arrow.

3.* For each of the numbers: 41, 83, 32, the first digit is greater in value than the second digit.

How many 2-digit numbers have this property?

4.* A cage contains birds and rabbits. There are sixteen heads and thirty-eight feet. How many birds are there in the cage?

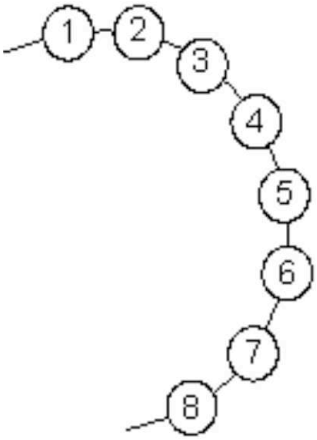
5.** Imagine writing down every possible combination of the digits 1, 2, 3, and 4. What would be the sum of all these combinations?

For example, with the digits 1, 2 and 3; there are six combinations and the sum of these combinations is 123 + 132 + 213 + 231 + 312 + 321 = 1332.

6. At a birthday party, one-half drank only lemonade, one-third drank only cola, fifteen people drank neither, and nobody drinks both.

How many people were at the party?

7.*A group of children stand holding hands in a large circle and a teacher walks around the circle giving each child in order a number 1, 2, 3, 4, ...



If number 12 is standing opposite number 30, how many children are there in the circle?

8. Alice, Belinda, and Clara were the three representatives for their school in a team cross country race. Alice finished the race in middle position, Belinda finished after Alice, in 19th position, and Clara finished 28th.

How many schools took part in the race?

Solutions

1. By writing the sums as: 1 + 999, 2 + 998, 3 + 997, ... , 499 + 501, 500 + 500. It is clear that the number of different pairs is five hundred.

How many different triplets of positive integers add to make one-thousand?

2. Consider the two diagrams below.

The area of the square is 4, so the area of the large triangle is 2 (half of the square) and the area of the small triangle is 1 (quarter of the square).

Hence the area of the arrow is 2-1 = 1 square unit.

3. If we begin to list the numbers in groups: 10; 20,21; 30,31,32; 40,41,42,43; ... ; 90,91,92,93,94,95,96,97,98 ; we can see that the total number of 2-digit numbers, for which the first digit is greater than the second digit, will be 1 + 2 + ... + 9 = 45.

4. If we let the number of birds be represented by b and the number of rabbits be represented by r then we get the following two equations:

$b + r = 16$ (1) $2b + 4r = 38$ (2)

Dividing the second equation by two gives: $b + 2r = 19$ (3)

If we now subtract equation (1) from equation (3) we get $r = 3$, and as $b + r = 16$ it follows that the number of birds, b, must be 13.

5. Considering combinations,

1234
1243
1324
1342
1423
1432
2134

Each digit appears in each column 6 times. So each column must add up to,
$6(1 + 2 + 3 + 4) = 60$.

Total = $60(1 + 10 + 100 + 100) = 66660$.

6. As $1/2 + 1/3 = 3/6 + 2/6 = 5/6$, we know that $1/6$ drank neither. So there must have been $6 \times 15 = 90$ people at the party.

7. Let c be the number of children in the circle. Half the children, $c/2$, will be in-between child 12 and child 30.

So $12 + c/2 = 30$. Therefore $c/2 = 18$.

That is, there are 36 children in the circle.

8. As Alice finished in middle position we know that there must be an odd number of runners in the race, but because there are three girls representing each school, the number of runners must be an odd multiple of three. In addition, we know that Clara, who finished 28th, was not last, as there are an odd number of runners. So there must be at least 29 runners.

If there were 33 runners, the middle position would be 17th. If there were 39 runners, the middle position would be 20th.

However, we are told that Belinda, who finished 19th, finished after Alice.

Hence there must have been 33 runners in the race and we deduce that eleven schools must have taken part.