Individual Problem #3 - *Narration*

"Take the numbers 0-9 and make a ten digit number out of them..."
Ok, this seems like another one of his tough ones. Let's just make the number the most simple it can be so that we get the idea. How about 9876543210.
"See if the first number is divisible by one". Duh! Obviously the first one is.
"See if the first two numbers are divisible by 2". Ok! So far so good.

As Peter read out each and every number, it felt like I just picked the winning lottery ticket number, each number working with the divisibility rule.

"See if the first seven numbers are divisible by seven". Doh! Ok, it's not as simple as it once appeared to be. Plus what are the chances of me picking out the right number. Let's see...1/10 x 1/9 x 1/8...WAIT! What am I doing?! There is a bigger problem here that is due next week. How do I tackle this problem?

Are there any numbers that **have** to be in a certain place. Certain divisibilities that force numbers to be somewhere.**Ten! Ten can only divide into numbers ending with a zero! The last number must be a zero**.So perhaps you have to work backwards in this problem and work out all the divisibility rules. What are they again?...Twos are evens...Threes?....hmm. I think I am supposed to know this to be a math teacher. Phew, before my own ignorance could really kick in Peter decides to go over the divisibility rules. Three does what now? Did anyone else understand what he said? I look around a notice quite a few people nodding their heads and a few claims of rediscovery. I'd better not ask for clarification...I'll just pack up my stuff and look it up later.

...Later...

Starbucks, with a coffee and working on math problems. Does it get any better than this? For starters, let's figure out those divisibility rules. Of course I can probably prove them to myself but why do that when someone else has taken the time of making a website filled with them.
www.google.com, key word search "divisibility rules"
Oh Dr. Math! Is there anything you don't know. From the website (http://mathforum.org/dr.math/faq/faq.divisibility.html) I get these rules:
"If the last digit is even, the number is divisible by 2.
If the sum of the digits is divisible by 3, the number is also.
If the last two digits form a number divisible by 4, the number is also.
If the last digit is a 5 or a 0, the number is divisible by 5.
If the number is divisible by both 3 and 2, it is also divisible by 6.
Take the last digit, double it, and subtract it from the rest of the number;
if the answer is divisible by 7 (including 0), then the number is also.
If the last three digits form a number divisible by 8, then so is the whole number.
If the sum of the digits is divisible by 9, the number is also.
If the number ends in 0, it is divisible by 10."

Some of them seem pretty specific while others seem like they could work with lots of different numbers. From the rules this is what we know...

- The last number is 0
- The 9th number could be anything but zero
- The 8th number could be anything, just so as the 6th, 7th and 8th are divisible by 8...and must be an even number
- The 7th number, if you double it and subtract it from the rest of the 1-6 must be divisible by 7
- The first 6 numbers must be both even and add to be divisible by 3
- **OOOh! The fifth number must be 5! Because the zero was already used!**
- The 3rd and 4th numbers must be divisible by 4
- The first three must add to be divisible by 3
- The second number must be even
- The first number can be anything.
- There are only a few places that the even numbers can go.

I consider working backwards but the problem with that is you have a large amount of numbers. The nice thing about working at the beginning is the numbers are small.

Let's consider the 3rd and 4th numbers
If the first three add to be divisible by 3 then there must only be so many combinations. The largest number possible would be 987, which adds to 24. Let's consider all the factors of 3 from 3-24 (remember, the zero and five have been used).

3 : No three numbers add to 3
6: 1+2+3
9: 1+2+6, 2+3+4
12: 1+2+9, 1+3+8, 1+4+7, 2+3+7, 2+4+6,
15: 1+6+8, 2+4+9, 2+6+7, 3+4+8....(seems like there should be more of those)
18: 1+8+9, 2+7+9, 3+6+9, 3+7+8, 4+6+8,
21: 4+8+9, 6+7+8
24: 9+8+7

Brief break. Is this really the easiest way to go about doing this. What if I missed a combination? It would create a hole in my logic. I am quickly reminded of the many sodoku puzzles I have butchered because I formulised (spell check tells me I made up that word) it and missed a combination. Back to work! Now, I must remember that for every set of numbers that add to a number divisible by three, there is another 5 combinations. The only thing I can rest assured of is that the second number has to be an even number. Lets see what combinations that produces.

Combinations divisible by three...
123, 321, 126, 162, 621, 261, 129, 921, 183, 381, 147, 741, 327, 723, 246, 264, 426, 624, 642, 462, 168, 186, 681, 861, 249, 429, 942, 924, 267, 627, 726, 762, 348, 384, 843, 484, 189, 981, 729, 927, 369, 963, 387, 783, 468, 486, 846, 648, 684, 864, 489, 849, 984, 948, 768, 786, 867, 687, 987

Oh great! I narrowed it down to 59 choices! Suddenly I get the feeling like I haven't really made much progress. Yet, I know if I just get one number, I can significantly narrow this list.

But wait! If there are only 4 spots available for even numbers and we have 4 even numbers to choose from, then we can't have an even number in an odd spot. The list can be narrowed down to.

123, 129, 147, 921, 183, 189, 381, 321, 369, 387, 327, 723, 741, 783, 729, 927,963, 987, 981...wow, that helps!

Let's try to narrow down the third and fourth number, maybe that way we can eliminate some of these choices....
The largest number the third and fourth can be is 98, so we find all combinations of the factors of four between 4-98 (that seems like a lot...but remember, we can't double numbers or use a zero or five...oh ya, and no even numbers in odd spots)
12, 16, 32, 36, 72, 76, 92, 96...doesn't narrow it down much. Although it created some probabilities that might help.

I am not sure where to go from here. We solved the 5th number, which means I should move on to 6 but that seems like a large task. I would have to find all the combinations that add to a number divisible by 3 and are even. Maybe I should go back to the random guess...wait...I did the calculations...1/362 880 chance I get it right. My odds don't seem favourable. Back to the drawing board...

First 6 must be divisible by 3 and even...
The largest the number it can be is 987654, which added is 39. The smallest it can be is 123456, which added is 21. We need 5 in the mix so lets look at the numbers that add to factors of 3 between 21-39
Since we know 5 is in every combination, we'll subtract 5 and just look for the other 5 numbers.
(21-5)16: 1+2+3+4+6
(24-5)19: 1+2+3+4+9, 1+2+3+6+7
(27-5)22: 9+7+1+3+2, 9+6+4+2+1, 8+7+4+2+1, 8+6+4+3+1, 7+6+4+3+2
(30-5)25: 1+3+4+8+9, 1+2+6+7+9, 2+3+4+7+9
(33-5)28: 9+8+7+3+1, 9+8+6+4+1, 9+8+6+3+2, 9+7+6+4+2,
(36-5) 31: 9+8+7+6+1, 9+8+7+4+3
(39-5) 34: 9+8+7+6+4

We must consider only combinations with the 2nd,4th and 5th numbers even (therefore, **we need 3 even numbers**). The 3rd and 4th numbers must be in our previous list of numbers (divisible by 4) and the last check is that the first three fall into our divisible by three numbers).

Combos that work:
987654, 947658, 789654, 749658...(it's easier than it appears. The set of numbers divisible by 4 can be significantly reduced to only numbers that include an odd and even number)...**981654, 941658, 189654, 149658, 983652, 923658, 389652, 329658,**

389256, 369258, 967254, 947256, 947652, 927654, 769254, 749256, 729654, 749652, 961254, 941256, 921654, 941652, 149256, 169254, 129654, 149652, 741258, 781254, 147258, 187254, 341658, 381654, 143658, 183654, 743256, 763254, 723654, 743652, 367254, 347256, 327654, 347652, 361254, 341256, 341652, 321654, 163254, 143256, 143652, 123654

46 combinations. Something tells me there is a much easier way to do this. However, I have done so much work that I can't help but to use the data I have acquired.

After crossing over with the combinations of the first 3 numbers, we only get the combinations of...
981654, 189654, 369258, 927654, 729654, 921654, 129654, 741258, 147258, 381654, 183654, 723654, 327654, 321654, 123654, 987654

Wow! Now we are getting somewhere! Using the divisibility by 7 rule, I could go through each one of these and see how many work. Of course, that means 48 possible combinations (using the first 6 numbers plus the zero, there are only 3 combinations for each number there). Actually, there will be less, since we know it has to be an odd number...so only 32 combinations.
Out of those combinations, these numbers work...
7296541, 9216543, 1296547, 1472583, 3816547, 3216549

Each of these combinations could only go with one other number when we introduce the 8th number, since it must be even and we have already used 3 of the 4 even numbers. These are the possibilities...**72965418, 92165438, 12965478, 14725836, 38165472, 32165498**
The last three numbers must be divisible by 8...let's give them a try...
Only one works! **38165472**

We know the last number must be a zero and the only number that remains for the 9th spot is 9, making **3816547290**

Let's put it to the test...
3/1 = 1 check
38/2 = 19 check
381/3 = 127 check
3816/4 = 954 check
38165/5 = 7633 check
381654/6 = 63 609 check
3816547/7 = 545 221 check
38165472/8 = 4 770 684 check
381654729/9 = 42 406 081 check
3816547290/10 = 381654729 check

Did I just do that in one sitting? As I glance up at the clock I realize that I have been sitting in Starbucks for the last 3.5 hours. What did I discover? The only number that works is **3816547290** and that I am addicted to puzzles.